

## Think

Why must like variables be grouped together?

To simplify by adding exponents, the bases must be the same.

### Problem 6 Simplifying Expressions With Rational Exponents

Simplify the expression  $(2a^{\frac{2}{3}} \cdot 3b^{\frac{1}{4}})(a^{\frac{1}{3}} \cdot 5b^{\frac{1}{2}})$ .

$$= (2 \cdot 3 \cdot 5)(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}})(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}) \quad \text{Commutative and associative properties of multiplication}$$

$$= 30(a^{\frac{2}{3}} \cdot a^{\frac{1}{3}})(b^{\frac{1}{4}} \cdot b^{\frac{1}{2}}) \quad \text{Simplify.}$$

$$= 30(a^{\frac{3}{3}})(b^{\frac{3}{4}}) \quad \text{Add exponents that have the same base.}$$

$$= 30ab^{\frac{3}{4}} \quad \text{Simplify.}$$

**Got It?** 6. Simplify each expression.

a.  $2c^{\frac{3}{5}} \cdot 2c^{\frac{1}{5}}$

b.  $n^{\frac{1}{3}} \cdot n^{\frac{4}{3}}$

c.  $(b^{\frac{2}{3}} \cdot c^{\frac{2}{5}})(b^{\frac{4}{3}} \cdot c^{\frac{9}{5}})$

d.  $(3j^{\frac{2}{3}} \cdot 7m^{\frac{1}{4}})(3j^{\frac{1}{6}} \cdot 7m^{\frac{3}{2}})$

## Lesson Check

### Do you know HOW?

- What is  $8^4 \cdot 8^8$  written using each base only once?
- What is the simplified form of  $2n^{\frac{2}{3}} \cdot 3n^{\frac{3}{4}}$ ?
- What is  $(3 \times 10^5)(8 \times 10^4)$  written in scientific notation?
- Measurement** The diameter of a penny is about  $1.9 \times 10^{-5}$  km. It would take about  $2.1 \times 10^9$  pennies placed end to end to circle the equator once. What is the approximate length of the equator?

### Do you UNDERSTAND?



- Writing** Can  $x^8 \cdot y^3$  be written as a single power? Explain your reasoning.
- Reasoning** Suppose  $a \times 10^m$  and  $b \times 10^n$  are two numbers in scientific notation. Is their product  $ab \times 10^{m+n}$  *always*, *sometimes*, or *never* a number in scientific notation? Justify your answer.
- Error Analysis** Your friend says  $4a^{\frac{1}{2}} \cdot 3a^{\frac{1}{5}} = 7a^{\frac{1}{7}}$ . Explain your friend's error. What is the correct answer?



## Practice and Problem-Solving Exercises



### A Practice

Rewrite each expression using each base only once.

8.  $7^3 \cdot 7^4$

9.  $(-6)^{12} \cdot (-6)^5 \cdot (-6)^2$

10.  $9^6 \cdot 9^{-4} \cdot 9^{-2}$

11.  $2^2 \cdot 2^7 \cdot 2^0$

12.  $5^{-2} \cdot 5^{-4} \cdot 5^8$

13.  $(-8)^5 \cdot (-8)^{-5}$

Simplify each expression.

14.  $m^3m^4$

15.  $5c^4 \cdot c^6$

16.  $4t^{-5} \cdot 2t^{-3}$

17.  $(x^5y^2)(x^{-6}y)$

18.  $(5x^5)(3y^6)(3x^2)$

19.  $-m^2 \cdot 4r^3 \cdot 12r^{-4} \cdot 5m$

Write each answer in scientific notation.

- STEM** 20. **Biology** A human body contains about  $2.7 \times 10^4$  microliters ( $\mu\text{L}$ ) of blood for each pound of body weight. Each microliter of blood contains about  $7 \times 10^4$  white blood cells. About how many white blood cells are in the body of a 140-lb person?

See Problem 1.

See Problem 2.

See Problem 3.

- STEM** 21. **Astronomy** The distance light travels in one second (one light-second) is about  $1.86 \times 10^5$  mi. Saturn is about 475 light-seconds from the sun. About how many miles from the sun is Saturn?

Simplify each expression.

22.  $8^{\frac{1}{3}}$

23.  $625^{\frac{1}{4}}$

24.  $1000^{\frac{1}{3}}$

See Problem 4.

Simplify each expression.

25.  $16^{\frac{3}{4}}$

26.  $9^{\frac{5}{2}}$

27.  $64^{\frac{7}{3}}$

See Problem 5.

Simplify each expression.

28.  $(8b^{\frac{2}{3}} \cdot 9t^{\frac{1}{5}})(8b^{\frac{5}{3}} \cdot 9t^{\frac{3}{5}})$

29.  $(7d^{\frac{3}{2}} \cdot 2g^{\frac{5}{6}})(2g^{\frac{3}{2}} \cdot 7d^{\frac{5}{6}})$

30.  $(4r^{\frac{2}{5}} \cdot 5s^{\frac{2}{7}})(5s^{\frac{5}{7}} \cdot 4r^{\frac{3}{5}})$

See Problem 6.

**B** Apply

Complete each equation.

31.  $5^2 \cdot 5^{\square} = 5^{11}$

32.  $m^{\square} \cdot m^{-4} = m^{-9}$

33.  $2^{\square} \cdot 2^{\frac{1}{2}} = 2^1$

34.  $a^{\square} \cdot a^4 = 1$

35.  $a^{\frac{2}{3}} \cdot a^{\square} = a^{\frac{5}{6}}$

36.  $x^3y^{\square} \cdot x^{\square} = y^2$

37. **Think About a Plan** A liter of water contains about  $3.35 \times 10^{25}$  molecules. The Mississippi River discharges about  $1.7 \times 10^7$  L of water every second. About how many molecules does the Mississippi River discharge every minute? Write your answer in scientific notation.

- How can you use unit analysis to help you find the answer?
- What properties can you use to make the calculation easier?

38. When you simplify an algebraic expression like  $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$ , you know that the bases of the expressions must be the same. You also need to rewrite the exponents so that they have a common denominator.

- Explain why you need to find the common denominator to simplify.
- Simplify the expression  $c^{\frac{3}{5}} \cdot c^{\frac{1}{2}}$ .

Simplify each expression. Write each answer in scientific notation.

39.  $(9 \times 10^7)(3 \times 10^{-16})$

40.  $(0.5 \times 10^{-6})(0.3 \times 10^{-2})$

41.  $(0.2 \times 10^5)(4 \times 10^{-12})$

- STEM** 42. **Chemistry** In chemistry, a *mole* is a unit of measure equal to  $6.02 \times 10^{23}$  atoms of a substance. The mass of a single neon atom is about  $3.35 \times 10^{-23}$  g. What is the mass of 2 moles of neon atoms? Write your answer in scientific notation.

Simplify each expression.

43.  $\frac{1}{a^4 \cdot a^{-3}}$

44.  $8m^{\frac{1}{3}}(m^{\frac{1}{3}} + 2)$

45.  $-4x^3(3x^3 - 10x)$

46. **Open-Ended** Write  $y^6$  as a product of two powers with the same base in four different ways. Use only positive exponents.
- Write  $y^6$  as a product of two powers with the same base in four different ways, using negative or zero exponents in each product.
  - Reasoning** How many ways can you write  $y^6$  as the product of two powers? Explain your reasoning.



**Challenge** Simplify each expression.

47.  $3^x \cdot 3^{2-x} \cdot 3^2$

48.  $2^n \cdot 2^{n+2} \cdot 2$

49.  $3^{\frac{1}{4}} \cdot 2^y \cdot 3^2 \cdot 2^x$

50.  $(a + b)^2(a + b)^{-3}$

51.  $(t + 3)^{\frac{4}{5}}(t + 3)^{\frac{2}{5}}$

52.  $5^{x+1} \cdot 5^{1-x}$

53. **Nature** A book shows an enlarged photo of a carpenter bee. A carpenter bee is about  $6 \times 10^{-3}$  m long. The photo is 13.5 cm long. About how many times as long as a carpenter bee is the photo?

**Standardized Test Prep**

SAT/ACT

54. What is the simplified form of  $(2x^{\frac{1}{2}}y^{\frac{2}{3}})(4x^{\frac{1}{4}}y^{\frac{5}{6}})$ ?

(A)  $6x^{\frac{1}{2}}y^{\frac{2}{3}}$

(B)  $6xy$

(C)  $8x^{\frac{1}{2}}y^{\frac{7}{6}}$

(D)  $8x^{\frac{3}{4}}y^{\frac{3}{2}}$

55. What is the  $x$ -intercept of the graph of  $5x - 3y = 30$ ?

(F)  $-10$

(G)  $-6$

(H)  $6$

(I)  $10$

56. At the Athens Olympics, the winning time for the women's 100-m hurdles was  $2.06 \times 10^{-1}$  min. Which number is another way to express this time in minutes?

(A)  $0.206$

(B)  $20.6$

(C)  $206 \times 10^1$

(D)  $206 \times 10^{-2}$

57. What is the solution of  $4x - 5 = 2x + 13$ ?

(F)  $3$

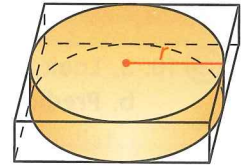
(G)  $4$

(H)  $9$

(I)  $32$

Extended Response

58. Bill's company packages its circular mirrors in boxes with square bottoms, as shown at the right. Show your work for each answer.



- What is an expression for the area of the bottom of the box?
- If the mirror has a radius of 4 in., what is the area of the bottom of the box?
- The area of the bottom of a second box is  $196 \text{ in.}^2$ . What is the diameter of the largest mirror the box can hold?

**Mixed Review**

Solve each system.

See Lesson 6-3.

59.  $2x + 3y = 12$   
 $-3x + y = -7$

60.  $2x - y = -3$   
 $x - y = 1$

61.  $2x + y = 15$   
 $-\frac{1}{2}x + y = 5$

Find the third, seventh, and tenth terms of the sequence described by each rule.

See Lesson 4-7.

62.  $A(n) = 10 + (n - 1)(4)$

63.  $A(n) = -5 + (n - 1)(2)$

64.  $A(n) = 1.2 + (n - 1)(-4)$

**Get Ready!** To prepare for Lesson 7-3, do Exercises 65–68.

Simplify each expression.

See Lesson 7-1.

65.  $(-2)^{-4}$

66.  $5xy^0$

67.  $4m^{-1}n^2$

68.  $-3x^{\frac{1}{3}}y^{-\frac{1}{2}}z^6$